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Финансовый пузырь как большое уклонение цен активов

Выступление
На семинаре НИУ ВШЭ
18 мая 2011 года

Financial Bubble as Large Asset Price Deviations

Sir Francis Bacon, *Novum Organum*

- “For whoever knows the ways of Nature will more easily notice its deviations; and, on
- the other hand, whoever knows its deviations will more accurately describe her ways.”

- «Тем, кому известны пути Природы, могут легко распознать её уклонения; напротив, тем, кому известны уклонения Природы, легко определить пути её развития.»

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Кризис 2007-09 гг: некоторые признаки

- **Кризис :** потери богатства на 25 трлн долларов, банковские списания на 2.0 трлн долларов, падение производства порядка 5-10 процентов и рост безработицы.

- **Особенность кризиса:** «провал» гигантских финансовых институтов.
- Важнейшая причина кризиса – *поведение крупных банков*
- {жадность, безрассудство и самонадеянность банкиров, FT. com.,Mar 12, 2009}
- **На антикризисные меры** в ведущих странах мира затрачено до 5 трлн долларов
- {FT. com., Apr. 3, 2009}

- **Модель нерегулируемого финансового рынка** показала свою неадекватность современным условиям.
- Heads we win, tails you lose

- **Идеология «рыночного фундаментализма»** оказалась серьёзно скомпрометированной

- Текущий кризис оказался **полной неожиданностью** для участников рынка, властей и науки.

Оценка потерь от кредитного кризиса за 2007-2009 гг возросла на два порядка

- **примерно $50 bn** (интервью *Б. Бернанке*, середина июля 2007 г).
- $100-$150 bn (заявление Федеральной Резервной системы, конец июля 2007 г)
- $400bn (заявление министров финансов G7 в Токио, февраль 2008 г)
- $945 bn (IMF, GFSR, апрель 2008 года)
- $1.4 trln (Bloomberg, March, 2009)
- **примерно $2.0 trln (Bloomberg, April, 2010)**

Дискуссии о новой экономической парадигме

- Most macroeconomics of the past 30 years was “spectacularly useless at best, and positively harmful at worst”
-
- ***P. Krugman***
- in the Lionel Robbins lecture
- at LSE, June 10th, 2009

Ортодоксия «рационального инвестора»

- Доктрина «эффективного рынка» и «репрезентативного рыночного агента» принципиально не допускает возможности провалов, кризисов и коллапсов. Могут ли оптимальные траектории вести к кризису?

- Финансовая практика никогда не воспринимала полностью теорию «эффективного» рынка (**У. Баффетт**, **Дж. Сорос**)

- Р.Лукас (2003): **«центральная задача макроэкономики – предотвращение депрессии практически решена»**

- **Финансовая система в целом не подобна «репрезентативному рыночному агенту», поскольку**
- А) происходят эффекты взаимодействия участников рынка;
- Б) агенты различаются масштабами, преференциями и информацией;
- В) крупные уклонения цен активов от фундаментальной стоимости заканчиваются кризисами.

Методология исследований «финансового пузыря»

- Идея ***Дж. М. Кейнса*** «животных инстинктов» (animal spirit) участников финансового рынка лежит в основе методологии современных исследований финансовых рынков.
- Гипотеза «долгового коллапса» (debt collapse) впервые высказана ***И. Фишером*** (Econometrica, 1933).
- Американский экономист ***Х. Минский*** в 80-90х годах разработал теорию долгового коллапса.

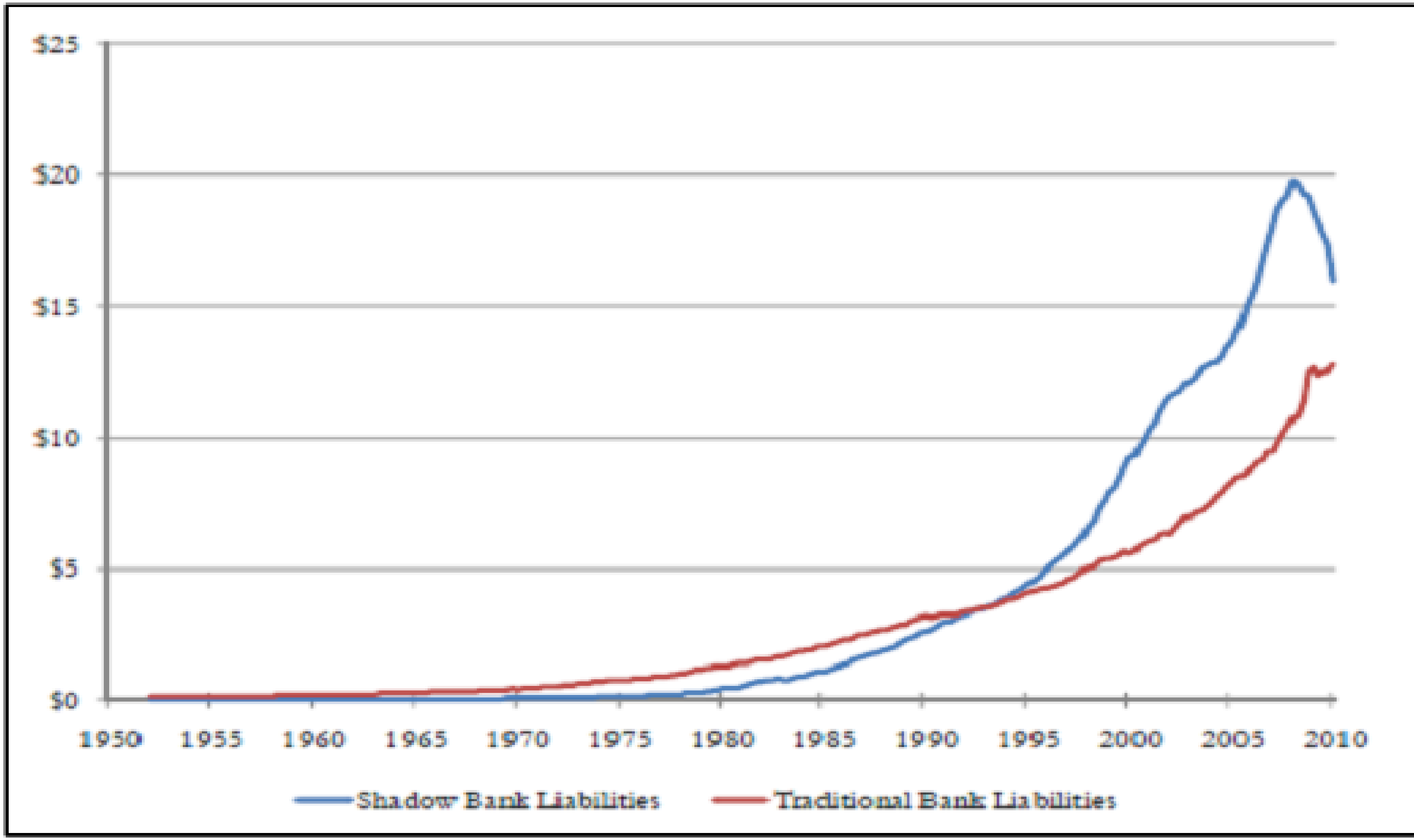
- Гипотеза «иррационального возбуждения» (irrational exeburance) ***Р. Шиллера*** и асимметричной информации ***Дж. Акерлофа***.
- ***Дж. Стиглиц*** о «новом экономическом мышлении»
- ***П.Кругмен*** и исследование экономических депрессий
- Концепция ***Р. Раджана*** (2005) качественных изменений финансовых систем

- Работы ***Б. Мандельброта*** о «фрактальных финансах».
- Эконофизика и модели перколации (percolation models) в изучении финансовых рынков (***Е. Стенли, Р. Мантенья, Д. Фармер, Д. Сорнет, А. Штауфер, Т. Лакс, Р. Конт, Ж-П. Бушад***).

Современная финансовая система

- Информационные технологии в финансах
- Дерегулирование финансовых рынков
- Новые финансовые институты (equity funds, venture capital firms, hedge funds, SIVs, etc)
- Формирование финансов на новой основе(more at arm's length system).
- «Устранение» финансового посредничества (desintermediation)
- Стимулирование рискованных операций и эффекты «подражания» (herding)
- Современные финансы имеют более глубокие и широкие рынки, лучше распределяют риски, но производят их в большем количестве; обеспеченность ликвидностью не возросла
- ***Систематическое уклонение рыночной стоимости активов от их «истинной» стоимости. Усиление склонности системы к появлению критических явлений (fat tails)***
- Большая интегрированность финансовых и реальных рынков увеличила вероятность глобальных кризисов

Традиционная и теневая банковские системы США
The Economist, Jul 13th,2010



Простая модель «финансового пузыря»

- Финансовым **кризисам** (кризисам ликвидности) всегда предшествовали «финансовые пузыри», или инфляционный рост стоимости активов.

  { С. *Киндлбергер, Б. Малкил, Х. Минский*}

- «Пузырь» раздувается по мере роста спроса на активы, а гетерогенный рынок продавцов и покупателей превращается в гомогенный рынок покупателей долга.
- Осознание невозможности всегда и всем *покупать* активы является сигналом для участников рынка к смене позиции с «длинной» на «короткую». Рынок покупателей сменяется рынком продавцов, т.е. происходит кризис ликвидности.

- **Кризис** есть результат качественных изменений в финансовой системе, происходящих вследствие

  автокаталитичности
- нелинейности
- сингулярности
- поведения участников финансового рынка.

- **Глобальный кризис (credit crunch) 2007-09 гг является процессом качественных изменений, перерождения финансового «пузыря».**

Постановка проблемы

- В рамках исследования **финансовых пузырей и кризисов** поставлена задача выяснения условий генезиса пузырей;
- «*Irrational Exuberance*”: отличия на ранних стадиях здорового роста конъюнктуры от «злокачественного» зарождения пузыря;
- *Р.Раджан* : «большие уклонения цен активов от их фундаментальной стоимости»

1. Financial market dynamics

Total assets, $A(t)$, are equal to the sum of money, $M(t)$, and the expected value of debt, $B(t)$:

(1) $$A(t) = M(t) + B(t),$$

where each variable is a continuous and at least twice differentiable function of time.

All the borrowers are to service their debt at the market rate of return, μ, subject to

$$dA = \mu\, B\, dt.$$

Creditors receive periodical (coupon) income, $dM = m(t)dt$, and acquire new debt, dB:

(2) $$\mu\, B(t)dt = m(t)dt + dB.$$

Comment: Applicability of the model to the ***QE*** process.

Given initial debt, $B(0)$, equation (2) can be solved with regard to the future debt value:

$$(3) \qquad B(t) = B(0)\exp[\mu\, t] - \int_0^t m(u)\exp[-\mu(u-t)]\,du.$$

The future debt value given by (3) might increase indefinitely in the future. Its amount

to be redeemed, $B(t) = 0$, gives the value of a continuously compounded annuity:

$$(4) \qquad B(0) = \int_0^t m(u)\exp[-\mu u]\,du.$$

By definition, the risk-adjusted rate of return, μ , is equal to the sum of

current yield, $\delta = \frac{m}{B}$, and the rate of capital appreciation (loss), $a = \frac{dB}{B}$:

$$(5) \quad \mu = \delta + a.$$

According to the CAPM theory, the risk-adjusted rate of return μ

could be decomposed into the sum of riskless rate, r , and risk premium, $\lambda\sigma$:

$$(6) \quad \mu = r + \lambda\sigma$$

where λ is a unit risk price.

The expected debt value is considered to be twice differentiable function of money issuance (density at any moment of time, s_t) alone, $B(t, s_t) = B(s_t)$.

Money issuance is supposed to be a random process depending upon time t, $s = s_t$:

(7) $\frac{ds}{s_t} = a\, dt + \sigma dz_t$

where a is drift and σ is volatility parameter for money issuance, s_t.

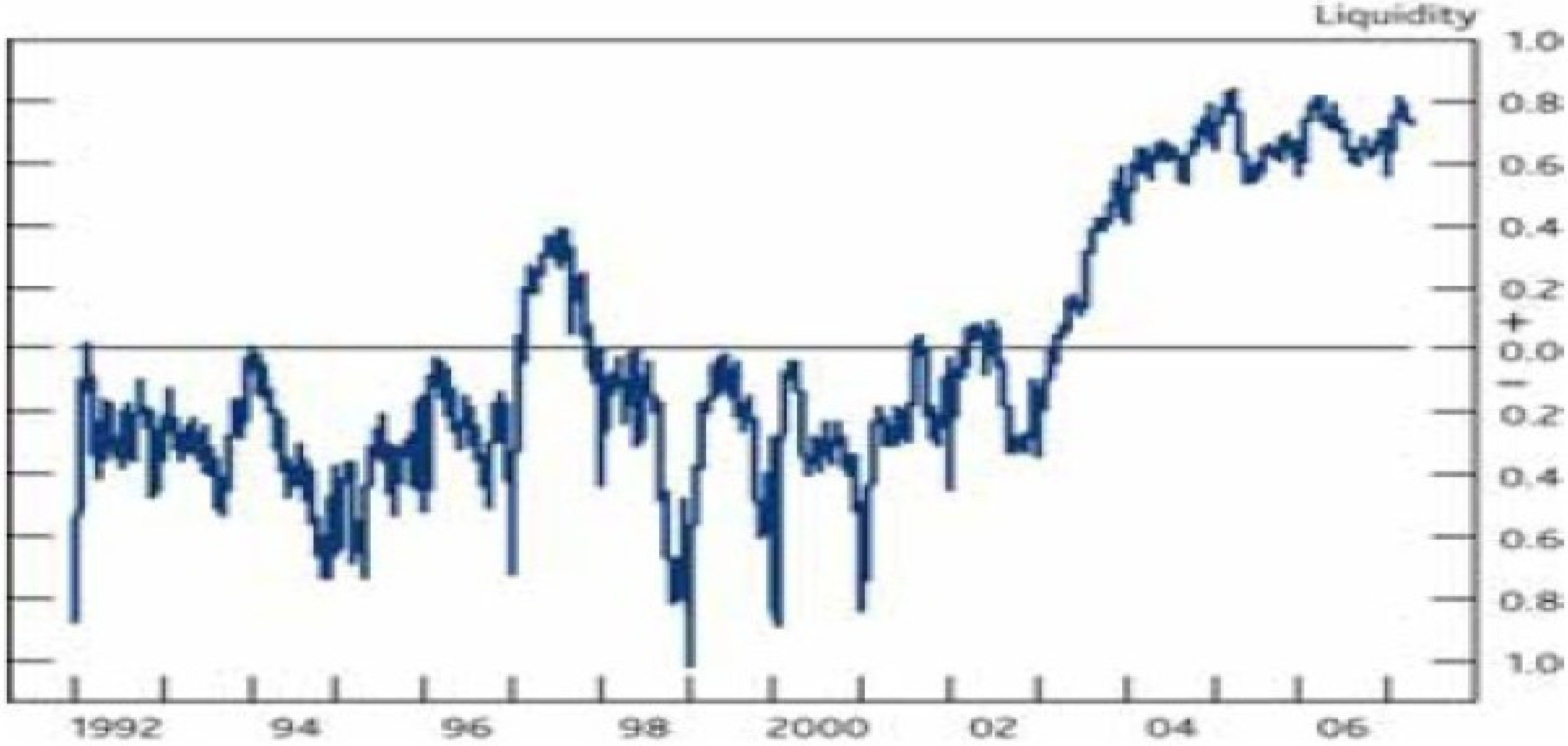
Stochastic differential equation (7) can be solved for money issuance:

(8) $s_t = s_0 \exp\left[(a - 0.5\sigma^2)t + \sigma z_t\right]$

where the term $z_t = \int_0^t dz_u$ is the Ito integral of random noise, and

(9) $\langle s_t \rangle = s_0 \exp[at]$

serves as a representation of the expected money issuance.



The weighted index of market liquidity measures for 1992-2006 (Gieve, 2006).

The central bank monetary policy, such as *quantitative easing, QE*, is performed in accordance with (8) subject to a stochastic noise while market participants expect money issuance in amounts given by (9):

$$(10) \quad dB = [\,\mu B(s_t) - s_t]dt + \sigma B(s_t)dz_t.$$

2. Options of new debt and debt protection

The expected debt value decomposition:

(11) $B(s_t) = F + [B(s_t) - F]$.

Since investors have no obligation to buy they have an option:

(12) $f(s_t) = [B(s_t) - F, 0]^+$

being written on the expected debt value $B(s_t)$ with par, F, as a strike price.

The market debt value a simple structurized product:

(13) $D(s_t) = B(s_t) - [B(s_t) - F, 0]^+$

The option to buy debt : $f(t) = [B(s) - F, 0]^+$

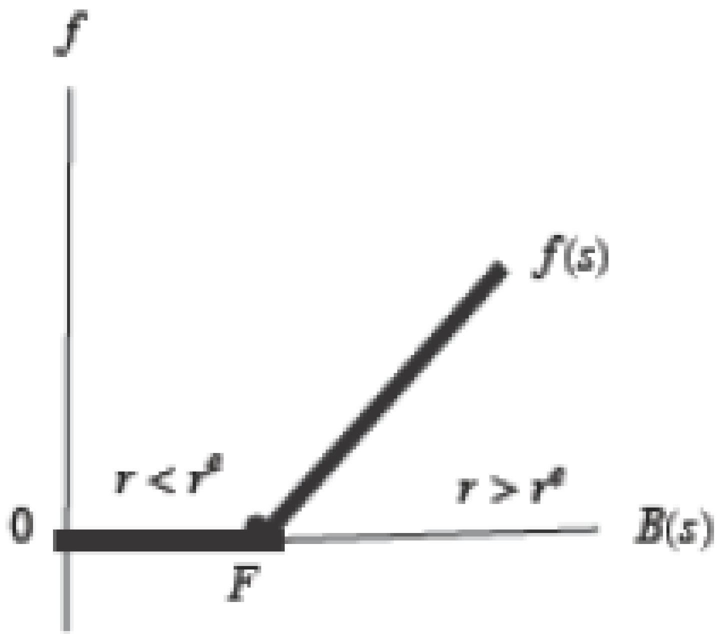


Figure 2. Matured option to purchase new debt.

Debt Guaranties

Bond holders protect their wealth by acquiring debt guaranties that are widely traded in financial markets. It follows (Merton, 1976) that the value of a debt guaranty has a put-to-default option representation:

$$(14) \quad P(s_t) = [F - B(s_t), 0]^+.$$

Market debt value is a simple structurized product of the following form:

$$(15) \quad D(s_t) = F - [F - B(s_t), 0]^+.$$

Market Debt and Assets Value

Equations (13) and (15) being taken together describe the market value of aggregate debt, $D(s_t)$:

$$(16)\quad B(s_t) - f(s_t) = D(s_t) = F - P(s_t).$$

Maturing option contracts for put and call values satisfy to the representation of total financial assets, $A(s_t)$:

$$(17)\quad B(s_t) + P(s_t) = A(s_t) = F + f(s_t).$$

Market Debt and Value of Capital

Due to the basic accounting equation

(18) $A(s_t) = D(s_t) + E(s_t)$,

the definition of the capital (equity) value takes place:

(19) $E(s_t) = f(s_t) + P(s_t)$,

where $E(s_t)$ is the value of the owner's capital for a financial system.

This value determines the Pigou effects upon the total wealth.

The structured debt : market debt value

$$D(s) = B(s) - f(s) = B(s) - [B(s) - F, 0]^{+}$$

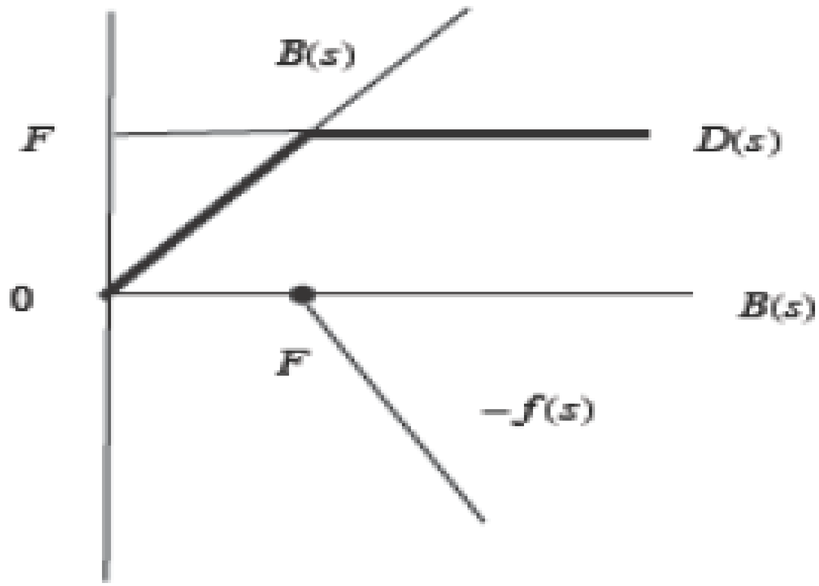


Figure 3. Market value of a debt at options maturity.

3. The expected debt valuation

The debt infinitesimal change due to the Ito lemma

(20) $dB = [as_t B'(s_t) + 0.5\sigma^2 {s_t}^2 B''(s_t)]dt + \sigma s_t B'(s_t) dz_t$

where debt derivatives are taken with respect to the liquidity issuance s_t. Since

(21) $r - \delta = a - \lambda\, \sigma$,

and, from (10) and (20),

(22) $\mu B(s_t) - s_t = \ as_t B'(s_t) + 0.5\sigma^2 {s_t}^2 B''(s_t)$

(23) $\sigma B(s_t) = \ \sigma s_t B'(s_t).$

Solution to the Expected Debt Value Equation

The inhomogeneous second order differential equation with respect to function $B(s_t)$:

(24) $0.5\sigma^2 s_t^2 B(s_t)'' + (r-\delta)s_t B(s_t)' - rB(s_t) + s_t = 0$.

which is an analogue to the well known Black-Sholes equation.

The expected debt value function $B(s_t)$

(25) $B(s_t) = B_1 s_t^{\beta_1} + B_2 s_t^{\beta_2} + \frac{1}{\delta} s_t,$

where $\beta_1 < 0$ and $\beta_2 > 1$ are real and distinct roots of the characteristic equation:

(26) $0.5\sigma^2 \beta(\beta - 1) + (r-\delta)\beta - r = 0.$

Expected Debt Value

Since $\beta_1 < 0$ the constant B_1 in (25) should be chosen as zero. This is so called "the absorption" condition requiring zero debt value. The second constant in is taken as zero since the expected debt equals to its fundamental value.

Hence the expected debt value, $B(s_t)$, becomes

(27) $$B(s_t) = \frac{1}{\delta} s_t.$$

Evaluation (at point $t = 0$) of the expected debt value:

(28) $$B_0 \equiv \langle B_0 \rangle = \int_0^\infty \langle s_t \rangle \exp(-\mu\, t)\, dt = s_0 \int_0^\infty \exp[-(\mu - a)]\, dt = \frac{1}{\delta} s_0.$$

4. Investors' new debt portfolio

Financial investors buy new and guarantee existing debt, and hedge their portfolios.

The portfolio, $\Phi(s_t)$, consisting of money issuance and new debt:

$$(29)\quad \Phi(s_t) = \theta_1 s_t + \theta_2 f(s_t),$$

where θ_1, θ_2 are the weights of new money and new debt, respectively.

This portfolio could be made riskless, if $\theta_1 = -f(s_t)'$ and $\theta_2 = 1$, and

$$(30)\, d\widehat{\Phi}(s_t) = 0.5\sigma^2 s_t^2 f''(s_t)dt.$$

The New Debt Value Equation

The riskless return on the hedged portfolio is less by the amount of

$\theta_1 \delta\, s_t dt$, which is lost due to hedging:

$$(31) \quad r[\theta_1 s_t + \theta_2 f(s_t) - \theta_1 \delta\, s_t]dt = 0.5\sigma^2 s_t^2 f''(s_t)dt\ .$$

Hence the riskless portfolio held by investors:

$$(32) \quad 0.5\sigma^2 s_t^2 f''(s_t) + (r-\delta)s_t f'(s_t) - rf(s_t) = 0\ .$$

The value of the option to buy new debt is a function:

$$(33) \quad f(s_t) = K_1 s_t^{\beta_1} + K_2 s_t^{\beta_2}.$$

The new debt value

The first constant in the r.h.s. of (33) has taken as zero, and the option to purchase new debt (33) becomes:

(34) $f(s_t) = Ks_t^{\beta}$

where $K \equiv K_1 > 0, \beta \equiv \beta_1 > 1$. Since call option is exercised in the money, investors, quite naturally, behave so as to maximize the value of option (34) under "easy money" policy or the "quantitative easing, QE".

5. Large asset prices deviation

In order to find point $s = s^*$ the second order differential equation (32) has to be complemented with three boundary conditions: the initial value condition, $f(0) = 0$, together with the value-matching condition

(35) $f(s^*) = F - B(s^*)$,

and the smooth-pasting condition

(36) $f(s^*)' = B(s^*)'$.

Optimal Money Issuance and Debt Value

Upon substitution of the expected debt value (27) and the new debt value (34)

into equations (35) and (36), point $s_t = s^*$ could be found as the following quantity:

(37) $$s^* = \frac{\beta}{\beta-1}\delta F.$$

The free boundary point $s = s^*$ delivers maximum value to the debt purchase option.

Money issuance at the free boundary point maximizes the expected value of debt,

$B(s)$, up to

(38) $$B(s^*) = F + f(s^*).$$

Due to put-call equivalence theorem (16) the put value would go to the zero,

$P(s^*) = 0$, while the market value of debt is bounded by its nominal value, $D(s^*) = F$.

Put-to-Default Option at the critical point

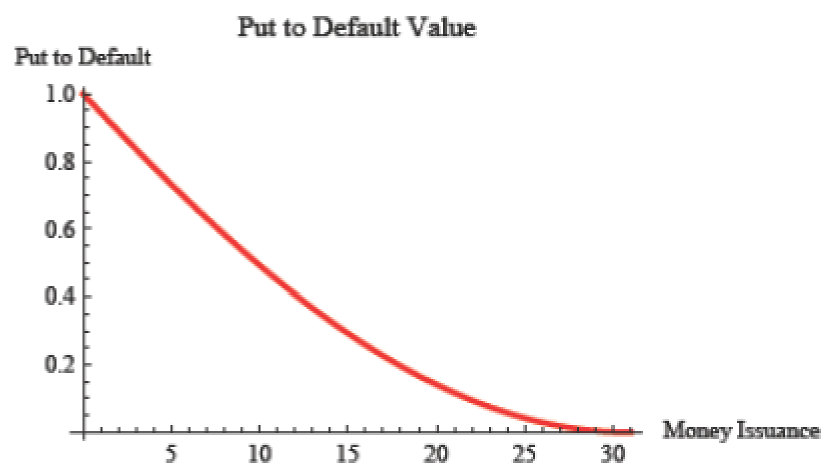


Figure 4. Value of debt protection.

6. Critical point without herding

To purchase new debt investors in aggregate have to spend their money.

Additional demand for new debt supports its growing price inducing investors to substitute market debt value for its expected value. These coherent actions imply a persistent process of the debt overvaluation at the critical point

(39) $$\frac{B(s^*)}{D(s^*)} = \frac{1}{\delta}\frac{\beta}{\beta-1}\delta F \colon F = \frac{\beta}{\beta-1} > 1,$$

where the magnitude $\frac{\beta}{\beta-1}$ defines *the scale of the asset prices divergence*.

Assets and Debt without Herding

Under "normal" conditions, as it follows from (17), the total assets value:

(40) $A(s^*) = F + f(s^*)$,

since, by definition, the market debt value at the critical point equals to its nominal value: $D(s^*) = F$.

What is the amount of financial equity in the system at the critical point?

The answer depends upon *the hypothesis of herding*.

Distance-to-Default

Assuming no herding at the critical point of liquidity issuance s^* rational investors would be keeping a nonzero value of their own capital:

(41) $E(s^*) = A(s^*) - D(s^*) = \frac{1}{\delta}\frac{\beta}{\beta-1}\delta F - F = \frac{1}{\beta}F = f(s^*) > 0.$

Hence the "distance-to-default" magnitude, or the system "survival" is

(42) $\Pr[survival \equiv Distance - to - Default] = \frac{A(s*)-D(s*)}{A(s*)} = 1 - \frac{\beta-1}{\beta} = \frac{1}{\beta}.$

Alternatively, the probability of financial default:

(43) $0 < Pr[default] = \frac{D(s^*)}{A(s^*)} < 1.$

Since $\Pr[default] = 1 - \Pr[survival]$, the default probability is:

(44) $\Pr[default] = \frac{\beta-1}{\beta}.$

7. Herding at critical point

The process of hedging in the model would imply that financial investors substitute the market debt value, $D(s_t)$, for its expected value, $B(s_t)$.

At the critical point, $s_t = s^*$, the amount of capital in the system diminishes virtually to the zero:

(45) $$E(s^*) = A(s^*) - B(s^*) = 0$$

which implies that *a posteriori* probability of default equals to one:

(46) $$\Pr\,[default] = \frac{B(s^*)}{A(s^*)} = 1\,,$$

that makes crisis to be a virtually inevitable event.

Leverage at the Critical Point

In spite of the finite scale of a bubble given by (39), the market leverage performed by investors, $D(s)/E(s)$, might grow indefinitely, since

$D(s^*) = A(s^*) = B(s^*)$. As a direct consequence of such a development, financial leverage ratio

$$\frac{A(s^*)}{E(s^*)} \to \infty, \quad (47)$$

It was studied extensively in (Adrian and Shin, 2008).

Leverage Singularity

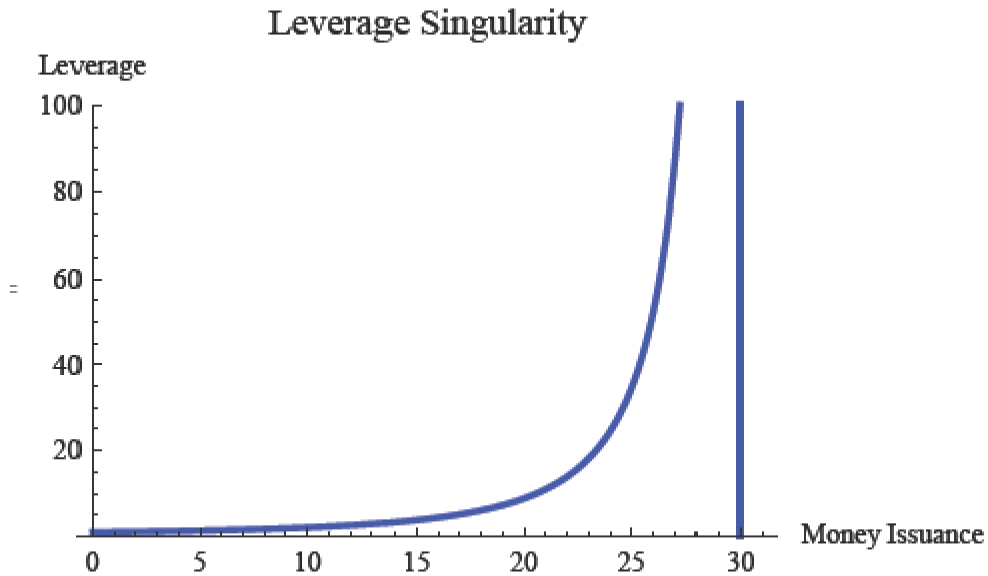


Figure 5. Leverage singularity.

Numerical Primer

8. Numerical validation

The model described above is validated numerically.

Nominal debt of 400 billion of dollars, $F = \$400bn$,

riskless rate of return per annum, $r = 0.05$,

annual risk-adjusted interest rate, $\mu = 0.07$; the current yield, $\delta = 0.045$,

and annual capital gain, $a = 0.025$. Amount of risks (per annum) in the system

is equal to $\sigma = 0.15$.

The characteristic equation of such a system:

$$0.5 \times \ 0.15^2 \beta(\beta - 1) + (0.05 - 0.045)\beta - 0.05 = 0$$

has two distinct real roots: $\beta_2 = -0.099$, and $\beta_1 = 2.404 > 1$

of which only the positive root has an economic meaning.

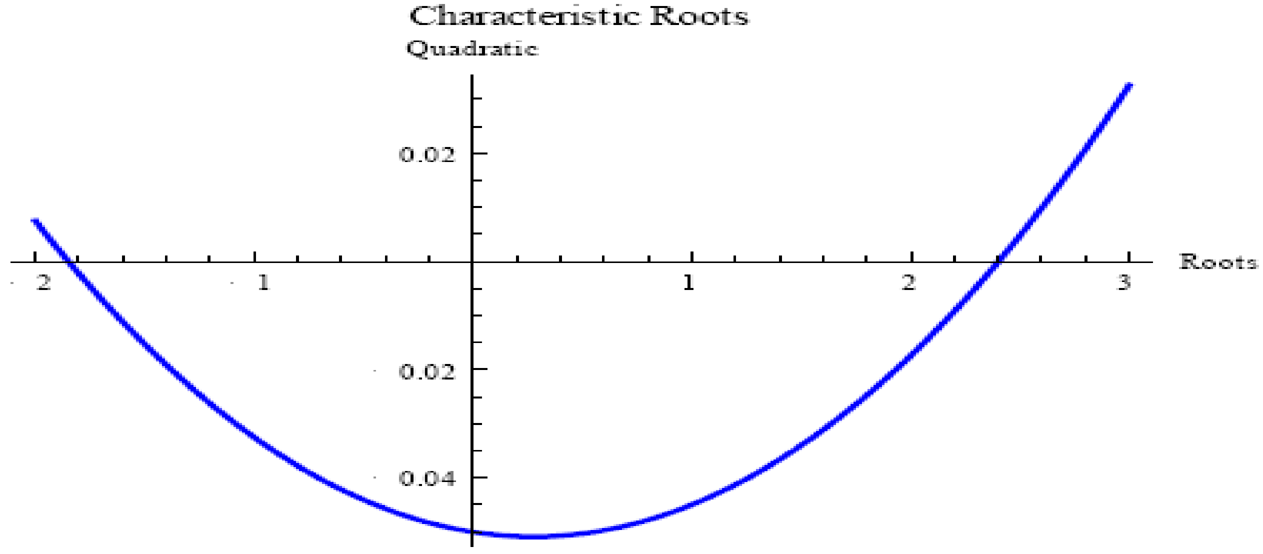


Figure 6. Characteristic equation and roots.

The critical point of money issuance being defined as in (32) is equal to

$$s^* = \frac{2.4}{1.4} \times 0.045 \times 400 = \$30.86bn.$$

This quantity defines the expected value of a debt at the critical point:

$$B(s^*) = \frac{1}{0.045} \times 30.86 = \$685.75bn,$$

and the value of new debt:

$$f(s^*) = 0.0761 \times 30.86^{2.4} = \$285.74bn$$

where constant $K = 0.0761$.

Total assets $A(s^*)$ are comprised of nominal debt, $F = \$400bn$, and equity, $E(s^*) = \$285.7bn$, totaling to \$685.7bn. The system survives with probability of $\Pr[DtD] = \frac{285.7}{685.75} = 0.42$. Alternatively, $\Pr[DtD] = \frac{1}{\beta} = 0.42$.

At the critical point with zero total equity, $E(s^*) = 0$, due to herding, the system default would have happened with probability of 1.0.

Bubble and Crisis

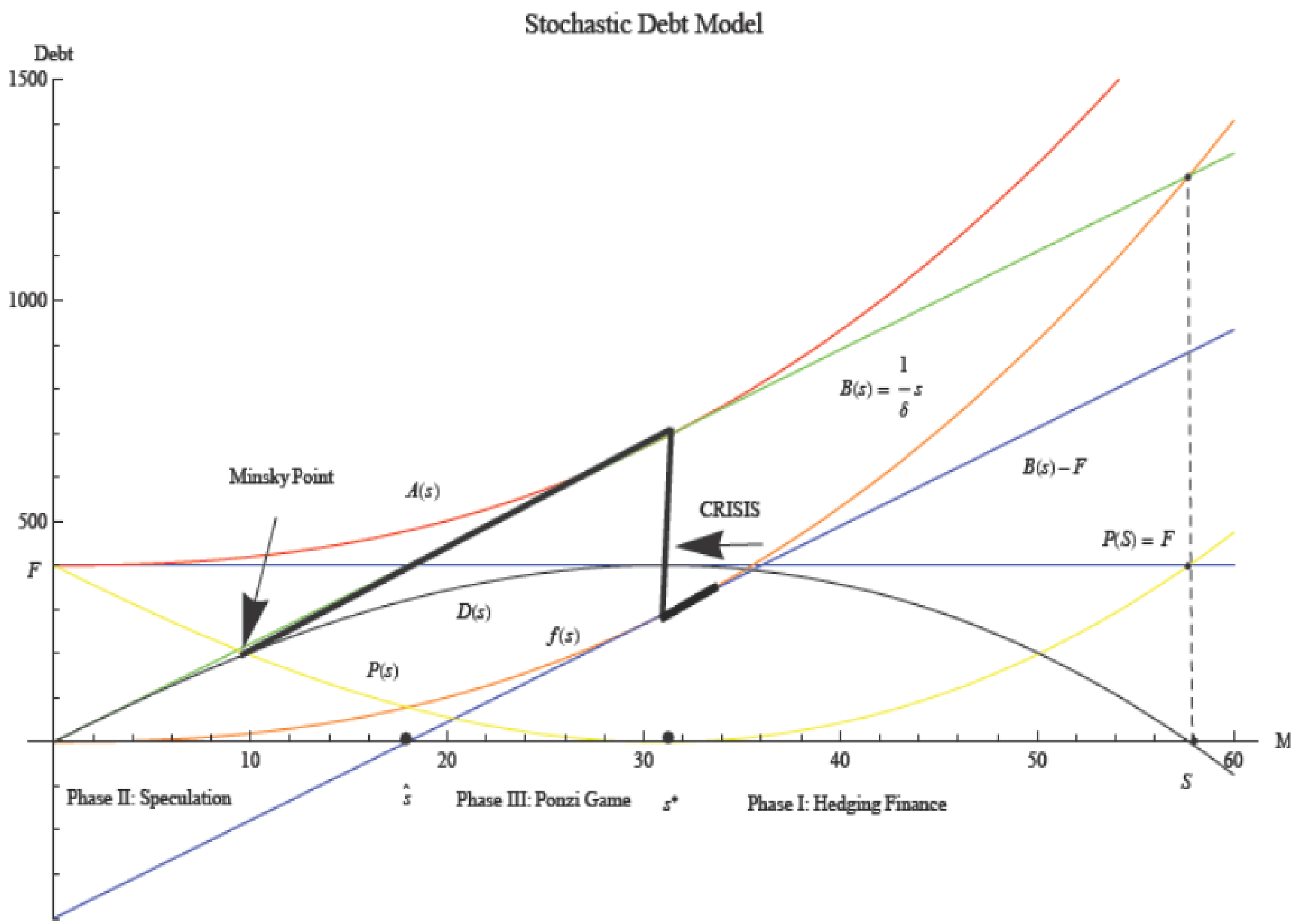


Figure 7. The Minsky phases in finance.

The Bubble Emergence

9. The Minsky point

It is important to distinguish between the trajectory of "normal" increases in the debt value $D(s_t)$, and the debt overvaluation process going on along trajectory $B(s_t)$. " The Minsky" point in the model is identified with the early detection of the asset price overvaluation. At this point the financial bubble emerges.

It could be calculated as the intersection between functions $D(s_t)$ or $B(s_t)$ with trajectory of the debt protection, $P(s_t)$. $s_M = \$10bn$

Herding and Bubble

10. Financial bubble singularity

The proposed model described just an emergence of a financial bubble by implying persistent substitution of the market debt value for its expected value. Bursting bubble could be represented via debt singularity that appears due to herding.

The simplest modification of this sort is the following:

(61) $f(s_t) = Ks_t^{\beta} + h * (s^* - s_t)^{-\gamma}$,

where the herding parameter is

$$h = \begin{cases} 1, if\ herding; \\ 0, if\ no\ herding; \end{cases}$$

and $\gamma = 2.39$ is one of the percolation invariant constants (Stauffer,2009).

The New Debt Market

Figure 8. The new debt function singularity.

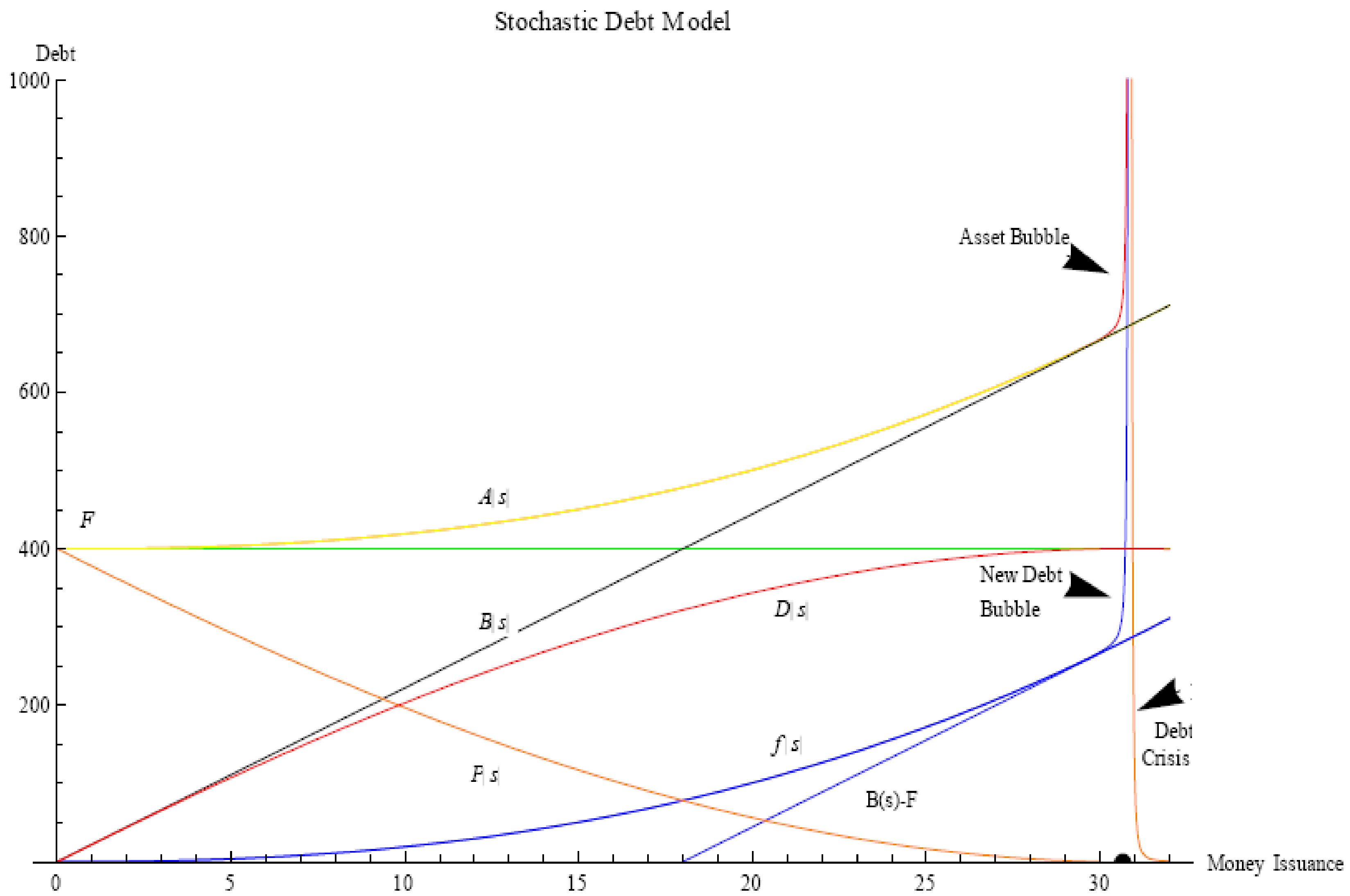


Figure 9. Financial bubble and crisis.

Financial Bubble and Percolation

In our numerical example the new debt function was taken as

$$f(s_t) = 0.076 s_t^{2.4} + h * (30.81 - s_t)^{-2.39} .$$

In the vicinity of the critical point the new debt function (61) is dominated entirely by its second component and thus (62) becomes

(62) $\frac{df}{ds} \sim (s^* - s_t)^{-\gamma-1}$

where(~) is the sign of asymptotic equality.

Некоторые выводы

- Механизм поведения финансовых инвесторов, ориентированный на «ожидаемую» стоимость – объективная основа возникновения кризисных явлений, включая коллапс системы
- «Большое уклонение» стоимости активов трансформируется в сингулярный процесс, если ускоряется рост стоимости активов под влиянием «эффекта толпы» (herding)
- Зарождение пузыря – точка Минского
- Финансовый пузырь – рост стоимости долга при возрастающем темпе роста стоимости. Пузырь - всегда сингулярен.